U S A Mathematical Talent Search

PROBLEMS

Round 3 - Year 10 - Academic Year 1998-99

1/3/10. Determine the leftmost three digits of the number

$$1^1 + 2^2 + 3^3 + \ldots + 999^{999} + 1000^{1000}.$$

2/3/10. There are infinitely many ordered pairs (m, n) of positive integers for which the sum

$$m + (m+1) + (m+2) + \ldots + (n-1) + n$$

is equal to the product mn. The four pairs with the smallest values of m are *(1, 1), (3, 6), (15, 35)*, and *(85, 204)*. Find three more *(m, n)* pairs.

3/3/10. The integers from 1 to 9 can be arranged into a 3×3 array (as shown on the right) so that the sum of the numbers in every row, column, and diagonal is a multiple of 9.

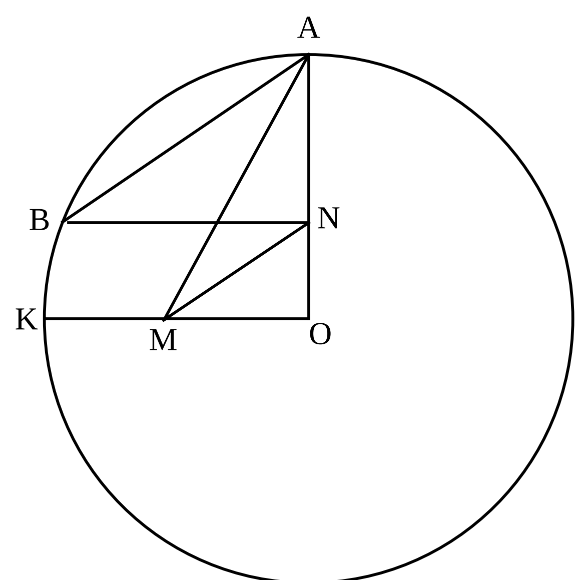
(a.) Prove that the number in the center of the array must be a multiple of 3.

(b.) Give an example of such an array with 6 in the center.

A	B	C
D	E	F
G	H	I

4/3/10. Prove that if $0 < x < \pi/2$, then $\sec^6 x + \csc^6 x + (\sec^6 x)(\csc^6 x) \geq 80$.

5/3/10. In the figure on the right, O is the center of the circle, OK and OA are perpendicular to one another, M is the midpoint of OK, BN is parallel to OK, and $\angle AMN = \angle NMO$. Determine the measure of $\angle ABN$ in degrees.



****

Complete, well-written solutions to **at least two** of the problems above, accompanied by a completed Cover Sheet, should be sent to the following address and **postmarked no later than January 9, 1999**. Each participant is expected to develop solutions without help from others.

USA Mathematical Talent Search
COMAP Inc., Suite 210
57 Bedford Street
Lexington, MA 02173